Lecture 10

Serviceability Criteria of the ACI Code for Deflection and Crack Width

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Objectives

At the end of this lecture, students will be able to

- Define short term and long term deflection
- Explain crack formation
- Identify relevant ACI codes to control deflection and crack width

Section 1: Deflections
Definition Effects

- It is important to maintain control of deflections so that members designed mainly for strength at prescribed overloads will also perform well in normal service.

- Excessive deflections can lead to cracking of supported walls and partitions, ill-fitting doors and windows, poor roof drainage, misalignment of sensitive machinery and equipment, and visually offensive sag etc.

Deflection Types

- Immediate (short-term)
  - Due to applied loads on the member.

- Long-term
  - Due to creep and shrinkage of concrete.
Deflection in RC One-way Slabs and Beams

• Immediate (short-term) Deflection
  
  • Immediate deflection at a given load in a structure is calculated using equations of elastic deflection.

  \[ \Delta = f(\text{loads, spans, supports})/EI \]

  • \( EI \) is the flexural rigidity.
  
  • \( f(\text{loads, spans, supports}) \) is a function of particular load, span and support arrangement.

Deflections can be directly computed for different conditions of loading and end conditions as below:

**Case 1**

\[ \Delta_{\text{mid}} = \frac{5}{384} \cdot \frac{wL^4}{EI} = \frac{5}{48} \cdot \frac{M_{\text{end}}^2}{EI} \]

**Case 2**

\[ \Delta_{\text{mid}} = \frac{1}{185} \cdot \frac{wL^4}{EI} = \frac{128}{1685} \cdot \frac{M_{\text{end}}^2}{EI} \]

**Case 3**

\[ \Delta_{\text{mid}} = \frac{1}{384} \cdot \frac{wL^4}{EI} = \frac{1}{16} \cdot \frac{M_{\text{end}}^2}{EI} \]

**Case 4**

\[ \Delta_{\text{mid}} = \frac{1}{192} \cdot \frac{Pb^3}{EI} = \frac{1}{24} \cdot \frac{M_{\text{end}}^2}{EI} \]
Deflection in RC One-way Slabs and Beams

- Immediate (short-term) Deflection
  
  **Determination of $l$:** In elastic deflection formulae, $l$ is the span length and is least of:
  
  - $l_n + h$
  - c/c distance between supports.

![Diagram of a beam](image)

- Determination of $E_c$ (modulus of elasticity of concrete)
  
  Unless stiffness values are obtained by a more comprehensive analysis, immediate deflection shall be computed with the modulus of elasticity ($E_c$) of section as given in ACI 19.2.2.1.

  For normal weight concrete  \( E_c = 57,000 \sqrt{f_{cc}} \) (psi)
Deflection in RC One-way Slabs and Beams

- Immediate (short-term) Deflection
  - Determination of $I$: In elastic deflection equation, the effective moment of inertia $I_e$ is calculated as (ACI 24.2.3.5a):

  $$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

  Where, $M_a = $ maximum service load moment at the stage for which deflections are being considered.
  $M_{cr} = $ Cracking moment = $f_r I_g / y_t$ (where $f_r = 7.5 \sqrt{f'c}$)
  $I_g = $ Gross moment of inertia
  $I_{cr} = $ Moment of inertia of cracked section

Deflection in RC One-way Slabs and Beams

- Long-term Deflections
  - Shrinkage and creep due to sustained loads cause additional long-term deflections over and above those which occur when loads are first placed on the structure.
  - Such deflections are influenced by:
    - Temperature,
    - Humidity,
    - Curing conditions,
    - Age at the time of loading,
    - Quantity of compression reinforcement, and
    - Magnitude of the sustained load.
Deflection in RC One-way Slabs and Beams

• Long-term Deflections
  • Additional Long-term deflection resulting from the combined effect of creep and shrinkage is determined by multiplying the immediate deflection caused by the sustained load with the factor $\lambda_A$ as given in ACI table 24.2.4.1.3.

\[ \Delta_{(cp+sh)} = \lambda_A \Delta_{s} \]

\[ \lambda_A = \frac{\zeta}{1 + 50\rho'} \]

\[ \rho' = \frac{A_s'}{bd} \]

<table>
<thead>
<tr>
<th>Sustained load duration, months</th>
<th>Time dependent factor, $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 months</td>
<td>1.0</td>
</tr>
<tr>
<td>6 months</td>
<td>1.2</td>
</tr>
<tr>
<td>12 months</td>
<td>1.4</td>
</tr>
<tr>
<td>60 or more</td>
<td>2.0</td>
</tr>
</tbody>
</table>

It is important to note here that long term deflections are function of immediate deflections due to sustained load only i.e.,

\[ \Delta_{(cp+sh)} = \lambda_A \Delta_{s} \]

• Sustained loads are loads that are permanently applied on the structure e.g., dead loads, superimposed dead loads and live loads kept on the structure for long period.
Deflection in RC One-way Slabs and Beams

- Deflection Control according to ACI code:
  - Two methods are given in the code for controlling deflections.
  - Deflections may be controlled directly by limiting computed deflections [ACI Table 24.2.2] or indirectly by means of minimum thickness ACI Table 7.3.1.1 and 9.3.1.1 for one-way systems.
  - In direct approach, the deflections are said to be within limits if the combined effect of immediate and long term deflections does not exceed the limits specified in ACI table 24.2.2.

Table 24.2.2—Maximum permissible calculated deflections

<table>
<thead>
<tr>
<th>Member</th>
<th>Condition</th>
<th>Deflection to be considered</th>
<th>Deflection Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat slabs</td>
<td>Not supporting or attached to non-rectangular elements likely to be damaged by large deflections</td>
<td>Immediate deflection due to maximum of L, S, and A</td>
<td>5/1800</td>
</tr>
<tr>
<td>Flat slabs</td>
<td></td>
<td>Immediate deflection due to L</td>
<td>5/2400</td>
</tr>
<tr>
<td>Floors</td>
<td>Supporting or attached to non-rectangular elements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roof or floors</td>
<td>Likely to be damaged by large deflections</td>
<td>That part of the total deflection occurring after attainment of non-rectangular elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load</td>
<td>5/1800</td>
</tr>
<tr>
<td>Roof or floors</td>
<td>Not likely to be damaged by large deflections</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Limit not intended to safeguard against towing. Paving shall be checked by calculations of deflections, including added deflections due to ponded water, and assuming time-dependent effects of sustained loads, causes, construction tolerances, and reliability of previously for damages.
2 Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflections calculated to occur before attainment of construction elements. The amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of materials similar to those being considered.
3 Limits shall be permitted to be approached if measures are taken to prevent damage to supported or attached elements.
4 Limits not meant to provide for non-rectangular elements.
Deflection in RC One-way Slabs and Beams

- Deflection Control according to ACI code:
  - The deflections may be controlled indirectly if the thickness of beams and one-way slabs are greater than the minimum requirements given in ACI table 7.3.1.1 and 9.3.1.1.

<table>
<thead>
<tr>
<th>Support condition</th>
<th>Minimum b (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported</td>
<td>24</td>
</tr>
<tr>
<td>One end continuous</td>
<td>24</td>
</tr>
<tr>
<td>Both ends continuous</td>
<td>23</td>
</tr>
<tr>
<td>Cantilever</td>
<td>20</td>
</tr>
</tbody>
</table>

- However, this method is applicable only to the cases of loadings and spans commonly experienced in buildings and cannot be used for unusually large values of loading and span.

Section 2: Cracking in RC Members
Crack Formation

- All RC beams crack, generally starting at loads well below service level, and possibly even prior to loading due to restrained shrinkage.
- In a well designed beam, flexural cracks are fine, so-called hairline cracks, almost invisible to a casual observer, and they permit little if any corrosion to the reinforcement.

As loads are gradually increased above the cracking load, both the number and width of cracks increase, and at service load level a maximum width of crack of about 0.016 inch (0.40 mm) is typical.

- If loads are further increased, crack widths increase further, although the number of cracks do not increase substantially.
- The limiting value of crack width both for interior and exterior exposures is taken as 0.016 inch.
ACI Code provision for crack control

- Crack width is controlled in the ACI Code by following:

\[ s \text{ (inches)} = \left( \frac{540}{f_s} \right) - 2.5c_c \ OR \ 12\left( \frac{36}{f_y} \right) \] (whichever is less)

The center-to-center spacing between the bars in a concrete section shall not exceed "s" as given by the above equation.

\( f_s \) is the bar stress in ksi under service condition.

\( c_c \) is the clear cover in inches from the nearest surface in tension to the surface of the flexural tension reinforcement.

ACI Code provision for crack control

- The ACI code permits \( f_s \) to be taken as 60 percent of the specified yield strength \( f_y \) (because seldom will be reinforcing bars stressed greater than 60 \% of \( f_y \) at service loads).

- The condition \( "f_s = 0.60f_y" \) is for full service load condition. For loading less than that, \( f_s \) shall be actually calculated.
Maximum Spacing Requirement

- Maximum spacing requirement corresponding to concrete cover as per ACI equation \( s = \left( \frac{540}{f_s} \right) - 2.5c_c \):

For various values of concrete cover and \( f_s \) of 24, 36 and 45 ksi, the maximum center-to-center spacing between the reinforcing bars closest to the surface of a tension member to control crack width can be plotted as shown.

References

- Design of Concrete Structures 14th Ed. by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)